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Non-relativistic string and D-branes on AdS$_5\times$S^5 from Semiclassical approximation

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ABSTRACT: We show that non-relativistic actions of string and D-branes on AdS$_5\times$S^5 can be reproduced as a semiclassical approximation of the string and D-brane actions around static 1/2 BPS configurations. This is contrastive to the Penrose limit. For example, the pp-wave string can be recaptured as a semiclassical approximation around a 1/2 BPS particle rotating at the velocity of light. We argue that small deformation of a straight Wilson line would give a composite operator in the gauge-theory side that corresponds to a semiclassical state of the non-relativistic string, according to the semiclassical interpretation.

KEYWORDS: AdS-CFT Correspondence, D-branes.

Contents

1. Introduction

AdS/CFT duality [1, 2] has now been widely accepted and continues to be an important laboratory to look for new aspects of gauge theories as well as string theory. The duality however has not been rigorously proved and it is obviously important to find a further support and confirm it furthermore. One of the difficulties is to solve type IIB string on $\mathrm{AdS}_5\times\mathrm{S}^5$ definitely. The Green-Schwarz action constructed by Metsaev and Tseytlin [3] is too complicated to quantize it. Although it is suspected to be quantum integrable since Bena, Polchinski and Roiban have pointed out the classical integrability of AdS superstring [4], the attempts for the quantization have not succeeded yet. Thus it would still be a nice direction to seek a soluble subsector.

Indeed, Berenstein, Maldacena and Nastase found a soluble sector of AdS/CFT [5] by using the Penrose limit [6]. The action of AdS superstring is reduced to the pp-wave string, which becomes a free massive theory on the flat world-sheet by taking a light-cone gauge [7] and so it is exactly solvable [8].

A new soluble sector has been recently proposed by Gomis, Gomis and Kamimura [9] with the so-called "non-relativistic limit" [10].[1] In this limit the AdS superstring is reduced

[1] For non-relativistic limit of D-branes in flat space and AdS space, see [11 – 14] and [15, 16], respectively.

to a free theory on the AdS_2 world-sheet with a static gauge and it is a basically solvable theory. The gauge theory counterpart has not been clarified yet though some observation have been given in [9].

The non-relativistic limit is quite similar to the Penrose limit (For the comparison, see table 1). This similarity leads to a speculation that the non-relativistic limit can also be described as a semiclassical limit in the same way as the Penrose limit [17]. The pp-wave string and Penrose limit are recaptured as a semiclassical approximation of the AdS-superstring around a BPS particle rotating around the greatest circle in the S^5 at the speed of light [18]. On the other hand, in this paper we will show that non-relativistic limit is nothing but a semiclassical approximation around a "static" configuration. Concretely, we will reproduce the non-relativistic actions of string and D-branes, which have been obtained in [9] and [19] respectively, from the semiclassical approximation. Here it should be noted that the non-relativistic string action is nothing but the semiclassical action expanding around a static AdS_2 world-sheet previously obtained by Drukker, Gross and Tseytlin [20], where a 1/2 BPS straight Wilson line [21] is inserted in the $\mathcal{N}$=4 SYM side. What we will show newly is the equivalence between AdS-brane actions in the non-relativistic limit and those in the semiclassical limit. This shows that a non-relativistic limit is nothing but a semiclassical limit around a static configuration.

The semiclassical interpretation of the non-relativistic limit leads us to argue the AdS/CFT dictionary in the non-relativistic limit with the help of the symmetry argument. Our argument is that a non-relativistic string state would correspond to small deformation of a 1/2 BPS straight Wilson line. Unfortunately, it would be difficult to check the dictionary by directly computing the anomalous dimension with perturbation theory because no large $U(1)_R$ charge is included unlike the BMN case [5] and the BMN scaling technique does not work at all.

The content of this paper is as follows: In section 2 we introduce the Dirac-Born-Infeld (DBI) actions of D-branes on $AdS_5\times S^5$. The classical solutions, around which the actions are expanded, are also discussed. In section 3 we reproduce the non-relativistic actions from the semiclassical approximation. First the bosonic fluctuations are considered for each of the branes. After that, the fermionic fluctuations are discussed since these are almost the same for all the branes. In section 4 we argue the corresponding gauge-theory side. Section 5 is devoted to a summary and discussions. Some conventions and notations are collected in appendix.

2. Setup

Let us introduce the D-brane actions (including the F-string case) on the $AdS_5\times S^5$ background and discuss static classical solutions around which the actions are expanded in a semiclassical method.

2.1 The actions of strings and D-branes on $AdS_5\times S^5$

The Dp-brane action [22] (see also [23]) is composed of the two parts, the Dirac-Born-Infeld

	pp-wave string	non-relativistic string
limit	Penrose limit	non-relativistic limit
gauge	light-cone	static
8 bosons	8 massive	3 massive & 5 massless
transverse sym.	SO(4)×SO(4)	SO(3)×SO(5)
world sheet	flat	AdS_2
classical sol.	rotating particle (1/2 BPS)	static AdS_2 (1/2 BPS)
sym. of sol.	U(1)	SL(2,R)
the vacuum op.	single trace op. $\mathrm{Tr}(Z^J)$	straight Wilson line

Table 1: pp-wave string vs. non-relativistic string

(DBI) part and the Wess-Zumino (WZ) part as follows:

$$S = S_{\mathrm{DBI}} + S_{\mathrm{WZ}} = T_p \int_{\Sigma} (\mathcal{L}_{\mathrm{DBI}} + \mathcal{L}_{\mathrm{WZ}}) \tag{2.1}$$

where $T_p = \left(g_s(2\pi)^p \alpha'^{\frac{p+1}{2}}\right)^{-1}$ is the tension of the Dp-brane.

The DBI action is given by[2]

$$\mathcal{L}_{\mathrm{DBI}} = \sqrt{s \det(g+\mathcal{F})}\, d^{p+1}\xi\,, \quad g_{ij} = \mathbf{L}_i^A \mathbf{L}_j^B \eta_{AB}\,, \quad \mathbf{L}_i^A = \partial_i Z^{\hat{M}} \mathbf{L}_{\hat{M}}^A\,, \tag{2.2}$$

where $s=-1$ for a Lorentzian brane while $s=1$ for a Euclidean brane. $\mathbf{L}^A$ and L are supervielbeins on $AdS_5 \times S^5$ written in terms of the superspace coordinates $Z^{\hat{M}} = (X^M, \theta^\alpha)$ (For the concrete forms see appendix). Then $\mathcal{F}$ is a modified field strength $\mathcal{F} = F - \mathcal{B}$. $F = dA$ is a gauge field strength and $\mathcal{B}$ is (the pullback on the worldvolume of) the NS-NS two-form

$$\mathcal{H} = d\mathcal{B} = i\mathbf{L}^A \bar{L}\Gamma_A \sigma L\,, \quad \mathcal{B} = 2i\int_0^1 dt\, \hat{\mathbf{L}}^A \hat{\bar{L}} \Gamma_A \sigma \theta \tag{2.3}$$

where the symbol "hat" implies $\hat{E} = E(t\theta)$. In the case that we want to consider a single F-string, the DBI part should be replaced by the Nambu-Goto (NG) action by turning off the flux as $S_{\mathrm{NG}} = S_{\mathrm{DBI}}|_{\mathcal{F}=0}$ and the tension should also be replaced by $T = (2\pi\alpha')^{-1}$.

The WZ part is characterized by a supersymmetric closed $(p+2)$-form h_{p+2}

$$h_{p+2} = d\mathcal{L}_{\mathrm{WZ}} = \sum_{n=0} \frac{1}{n!} h^{(p+2-2n)} \mathcal{F}^n \tag{2.4}$$

with

$$h^{(\ell+2)} = \frac{\sqrt{s}}{\ell!}\left[\mathbf{L}^{A_1}\cdots\mathbf{L}^{A_\ell}\bar{L}\Gamma_{A_1\cdots A_\ell}\varrho L + \delta_{\ell,3}\frac{i\alpha}{5}(\epsilon_{a_1\cdots a_5}\mathbf{L}^{a_1}\cdots\mathbf{L}^{a_5} - \epsilon_{a'_1\cdots a'_5}\mathbf{L}^{a'_1}\cdots\mathbf{L}^{a'_5})\right] \tag{2.5}$$

[2]We suppressed the dilaton and axion factors here.

where $\varrho = (\sigma)^{-\frac{p-3}{2}} i\sigma_2$ with $\sigma = \sigma_3$ and $-\sigma_1$ for Dp-brane and F-string, respectively. The $(p+1)$-dimensional form of the WZ term is

$$\begin{aligned} \mathcal{L}_{\rm WZ} &= \int_0^1 dt \left[\mathscr{C} \wedge \mathrm{e}^{\hat{\mathcal{F}}} \right]_{p+1} + C^{(p+1)} \,, \\ \mathscr{C} &= \bigoplus_n \frac{2\sqrt{s}}{(2n-1)!} \hat{\mathbf{L}}^{A_1} \cdots \hat{\mathbf{L}}^{A_{2n-1}} \hat{\bar{L}} \Gamma_{A_1 \cdots A_{2n-1}} (\sigma)^n i\sigma_2 \theta \end{aligned} \tag{2.6}$$

where $C^{(p+1)}$ is a bosonic $(p+1)$-form satisfying $h_{p+2}\big|_{\rm bosonic} = dC^{(p+1)}$.

2.2 Classical static solutions

Here let us derive static D-brane solutions.

First we shall introduce some variation formulae for the supervielbeins and super spin connections, which can be derived from the MC equations give in (A.6). The formulae are as follows:

$$\begin{aligned} \delta \mathbf{L}^A &= d\delta x^A - \eta_{BC} \delta x^{AB} \mathbf{L}^C + \eta_{BC} \mathbf{L}^{AB} \delta x^C - 2i \bar{L} \Gamma^A \delta\theta \,, \\ \delta L &= d\delta\theta - \frac{\alpha}{2} \delta x^A \widehat{\Gamma}_A i\sigma_2 L + \frac{\alpha}{2} \mathbf{L}^A \widehat{\Gamma}_A i\sigma_2 \delta\theta - \frac{1}{4} \delta x^{AB} \Gamma_{AB} L + \frac{1}{4} \mathbf{L}^{AB} \Gamma_{AB} \delta\theta \,, \\ \delta \mathbf{L}^{ab} &= d\delta x^{ab} + 2\alpha^2 \mathbf{L}^a \delta x^b + 2\eta_{cd} \mathbf{L}^{ac} \delta x^{db} + 2i\alpha \bar{L} \widehat{\Gamma}^{ab} i\sigma_2 \delta\theta \,, \\ \delta \mathbf{L}^{a'b'} &= d\delta x^{a'b'} - 2\alpha^2 \mathbf{L}^{a'} \delta x^{b'} + 2\eta_{c'd'} \mathbf{L}^{a'c'} \delta x^{d'b'} + 2i\alpha \bar{L} \widehat{\Gamma}^{a'b'} i\sigma_2 \delta\theta \,, \end{aligned} \tag{2.7}$$

where we have introduced the following quantities:

$$\delta x^A = \delta Z^{\hat{M}} \mathbf{L}^A_{\hat{M}} \,, \quad \delta x^{AB} = \delta Z^{\hat{M}} \mathbf{L}^{AB}_{\hat{M}} \,, \quad \delta\theta^\alpha = \delta Z^{\hat{M}} L^\alpha_{\hat{M}} \,. \tag{2.8}$$

It also follows from (2.3) that

$$\delta \mathcal{B} = i\delta x^A \bar{L} \Gamma_A \sigma L + 2i \mathbf{L}^A \bar{L} \Gamma_A \sigma L + d\left(\int_0^1 dt \, [\delta\hat{x}^A \hat{\bar{L}} \Gamma_A \sigma\theta - \hat{\mathbf{L}}^A \delta\hat{\bar{\theta}} \Gamma_A \sigma\theta] \right) \,. \tag{2.9}$$

Then, by using the variation formulae (2.7) and (2.9), a variation of the Dp-brane action is derived as

$$\begin{aligned} \delta S &= T_p \int_\Sigma d^{p+1}\xi \, \frac{1}{2} \sqrt{s \det(g + \mathcal{F})} (g + \mathcal{F})^{ji} (\delta g + \delta F - \delta\mathcal{B})_{ij} \\ &\quad + T_p \int_B \sum_{n=0} \left[\frac{1}{n!} \delta h^{(p+2-2n)} \mathcal{F}^n + \frac{1}{(n-1)!} h^{(p+2-2n)} \mathcal{F}^{n-1} \delta\mathcal{F} \right] \,, \end{aligned} \tag{2.10}$$

where $\partial B = \Sigma$. For a classical solution this variation should be zero.

It is the standard procedure to impose some ansatz in finding a classical solution. First let us impose that $\theta = 0$. Then (2.10) reduces to

$$\begin{aligned} \delta S = {}& T_p \int_\Sigma d^{p+1}\xi \frac{1}{2} \sqrt{s \det(g_B + F)} (g_B + F)^{ji} \left(2e_i^A \eta_{AB} \delta \mathbf{L}^B_j |_{\theta=0} + \delta F_{ij} \right) \\ & + T_p \int_B \frac{\sqrt{s} i\alpha}{3!} \Bigg[\left(\epsilon_{a_1 \cdots a_5} \delta \mathbf{L}^{a_1} |_{\theta=0} e^{a_2} \cdots e^{a_5} - \epsilon_{a'_1 \cdots a'_5} \delta \mathbf{L}^{a'_1} |_{\theta=0} e^{a'_2} \cdots e^{a'_5} \right) \frac{1}{(\frac{p-3}{2})!} F^{\frac{p-3}{2}} \\ & \qquad + \frac{1}{5} \left(\epsilon_{a_1 \cdots a_5} e^{a_1} \cdots e^{a_5} - \epsilon_{a'_1 \cdots a'_5} e^{a'_1} \cdots e^{a'_5} \right) \frac{1}{(\frac{p-5}{2})!} F^{\frac{p-5}{2}} d\delta A \Bigg] \,, \end{aligned}$$

where $g_{Bij} = e_i^A e_j^B \eta_{AB}$ and $\delta \mathbf{L}^A|_{\theta=0} = d\delta x^A - \eta_{BC}\delta x^{AB} e^C + \eta_{BC}\omega^{AB}\delta x^C$.

Furthermore we require that $A_i = 0$. Then it further reduces to

$$\begin{aligned}\delta S = &-T_p \int_\Sigma d^{p+1}\xi \sqrt{s \det g_B} g_B^{ji} \left(\nabla_i e_j^A + \omega_i^A{}_B e_j^B\right) \delta x_A \\ &+T_p \int_B \frac{\sqrt{s} i\alpha}{3!} \Bigg[\delta_{p,3} \Big(\epsilon_{a_1\cdots a_5} (d\delta x^{a_1} + \eta_{bc}\omega^{a_1 b}\delta x^c)\, e^{a_2}\cdots e^{a_5} \\ &\qquad -\epsilon_{a_1'\cdots a_5'} (d\delta x^{a_1'} + \eta_{b'c'}\omega^{a_1' b'}\delta x^{c'})\, e^{a_2'}\cdots e^{a_5'} \Big) \\ &\qquad + \frac{1}{5}\delta_{p,5} (\epsilon_{a_1\cdots a_5} e^{a_1} e^{a_2}\cdots e^{a_5} - \epsilon_{a_1'\cdots a_5'} e^{a_1'} e^{a_2'}\cdots e^{a_5'}) d\delta A \Bigg],\end{aligned}$$

where we have partially integrated. The terms proportional to $\delta_{p,3}$ and $\delta_{p,5}$ can be deleted by performing a partial integration and using the relation $de^A = -\eta_{BC}\omega^{AB}e^C$. This is a consequence of $dh_{p+2} = 0$. Then the remaining term (the first line) vanishes when the following relation is satisfied:

$$\nabla_i e_j^A + \omega_i^A{}_B e_j^B = 0. \tag{2.11}$$

For the static configuration $X^{M_i} = \xi^i$ $(i = 0, \cdots, p)$ this relation can be derived as the pull-back of the following relation to the world-sheet Σ:

$$\nabla_M e_N^A + \omega_{MB}^A e_N^B = 0, \tag{2.12}$$

which follows from the definition of the spin connection $\omega_{MB}^A = -e_B^P \nabla_M e_P^A$. Thus the static gauge configuration is a trivial classical solution.

Besides it, one can find a generalized configuration represented by

$$X^{M_i} = \begin{cases} X^{M_i}(\xi^i) & \text{for } i = 0, \cdots, p \\ 0 & \text{for } i = p+1, \cdots, 9 \end{cases}. \tag{2.13}$$

One can easily check that this solves (2.11) as follows. First we rewrite (2.11) as

$$(\nabla_i \nabla_j X^M + \nabla_i X^P \nabla_j X^Q \Gamma_{PQ}^M) e_M^A = 0 \tag{2.14}$$

where we have used (2.12). Defining $\eta_i^M \equiv \partial_i X^M$ $(M \in \{M_i | i = 0, \cdots, p\})$ which is invertible for the classical solution (2.13) one derives by using $g^{kl}\eta_k^M \eta_l^N = G^{MN}$

$$\begin{aligned}\gamma_{ij}^k \partial_k X^M &= (g^{kl}\eta_l^P \partial_{(i}\eta_{j)}^Q G_{PQ} + g^{kl}\eta_i^P \eta_j^Q \eta_l^R G_{RS}\Gamma_{PQ}^S)\eta_k^M \\ &= \partial_i\partial_j X^M + \partial_i X^P \partial_j X^Q \Gamma_{PQ}^M\end{aligned} \tag{2.15}$$

which solves (2.14). Thus (2.13) is also a static Dp-brane solution but its parametrization of the world-volume coordinates is slightly generalized. Even for the F-string case the same solution can be derived. Hereafter we will concentrate on the solution (2.13).

The value of the classical action can be evaluated by putting the classical solution into the action as follows:

$$\mathcal{L}_c = \sqrt{s \det g_0}\, d^{p+1}\xi = \det(e_0)_i^A d^{p+1}\xi = \frac{1}{2}\epsilon_{A_0\cdots A_p} e_0^{A_0}\cdots e_0^{A_p}, \tag{2.16}$$

1-brane	3-brane	5-brane	7-brane
(2,0), (0,2)	(3,1), (1,3)	(4,2), (2,4)	(5,3), (3,5)

Table 2: The possible configurations of 1/2 BPS branes in $\mathrm{AdS}_5\times\mathrm{S}^5$

where $g_0 = g|_{\text{classical}}$ and $e_0^A = e^A|_{\text{classical}}$. The contribution (2.16) can be canceled out by adding $C^{(p+1)}$ of the form

$$C_0^{(p+1)} = -\frac{1}{2}\epsilon_{A_0\cdots A_p} e_0^{A_0}\cdots e_0^{A_p}\,,$$

only if one can find such a closed form that $h_{p+2}|_{\text{classical}} = dC_0^{(p+1)}$. For the 1/2 BPS D-branes we can easily show that

$$h_{p+2}|_{\text{classical}} = 0$$

and that $dC_0^{(p+1)} = 0$, hence the classical action (2.16) can be canceled out by the flux term. This cancellation is actually necessary for the consistent semiclassical approximation. For the semiclassical approximation we need to take a large tension limit, which surely corresponds to the limit operation in the non-relativistic limit. Thus, if the cancellation does not occur, then the classical action diverges. That is why the cancellation is necessary.

It is helpful to remember a classification of possible 1/2 BPS configurations of D-branes on $\mathrm{AdS}_5\times\mathrm{S}^5$ (listed in table 2) [24, 25]. The notation of (m,n)-brane implies a brane configuration whose worldvolume extends along m directions in the AdS_5 and n directions in the S^5. Such a restriction against the directions has its roots in a 1/4 BPS intersecting condition before taking the near-horizon limit [24]. The classification could be reproduced in various ways, brane probe analysis [24], κ-invariance [25] and consistent non-relativistic limit [19] or semiclassical limit as discussed here.

3. Quantum fluctuations around the static solutions

From now on we shall consider a semiclassical approximation of the D-brane action around the classical solutions obtained in the previous section. As a result we see that the non-relativistic D-brane actions are reproduced.

First of all, let us expand the D-brane action in terms of the fermionic variables θ:

$$\mathcal{L}_{\text{DBI}} = \sqrt{s\det(g_B+F)}\left[1+\frac{1}{2}(g_B+F)^{ji}(g_F-B_F)_{ij}\right]d^{p+1}\xi + O(\theta^4)\,,$$

$$\mathcal{L}_{\text{WZ}} = C^{(p+1)} + \sum_{n=1}\frac{-\sqrt{s}}{(2n-1)!}e^{A_1}\cdots e^{A_{2n-1}}\bar\theta\Gamma_{A_1\cdots A_{2n-1}}(\sigma)^n i\sigma_2 D\theta\frac{F^{p+1-2n}}{(p+1-2n)!} + O(\theta^4)\,,$$

where g_B , g_F and B_F are

$$g_{Bij} = e_i^A e_j^B\eta_{AB}\,,\quad g_{Fij} = 2ie^A_{(i}\bar\theta\Gamma^B D_{j)}\theta\eta_{AB}\,,\quad B_F = -ie^A\bar\theta\Gamma_A\sigma D\theta\,.$$

Then the bosonic and the fermionic parts are given by

$$S_B = T_p \int d^{p+1}d\xi \sqrt{s\det(g_B+F)} + T_p \int_\Sigma C^{(p+1)} \,,$$

$$S_F = T_p \int d^{p+1}d\xi \sqrt{s\det(g_B+F)}\frac{1}{2}(g_B+F)^{ji}(g_F-B_F)_{ij}$$

$$+T_p \int_\Sigma \sum_{n=1} \frac{-\sqrt{s}}{(2n-1)!} e^{A_1}\cdots e^{A_{2n-1}}\bar{\theta}\Gamma_{A_1\cdots A_{2n-1}}(\sigma)^n i\sigma_2 D\theta \frac{F^{p+1-2n}}{(p+1-2n)!} + O(\theta^4) \,.$$

Our purpose is to examine the fluctuations around the classical solution. Hence let us decompose the variables into the fluctuations (expressed with "tilde") and the background (2.13) with $\theta = 0$ and $A_i = 0$ as follows:

$$X^{\underline{M}_i} = X_0^{\underline{M}_i}(\xi^i) + \tilde{X}^{\underline{M}_i} \,, \quad X^{\bar{M}_i} = 0 + \tilde{X}^{\bar{M}_i} \,, \quad \theta = 0 + \tilde{\theta} \,, \quad A_i = 0 + \tilde{A}_i \,,$$

where $\underline{M} = \{\, M_i \mid i = 0, \cdots, p \,\}$ and $\bar{M} = \{\, M_i \mid i = p+1, \cdots, 9 \,\}$.

Here we should remark on the treatment of the longitudinal fluctuations. In order to maintain the static gauge form (2.13), we have to restrict the longitudinal fluctuation $\tilde{X}^{\underline{M}_i}$ to be $\tilde{X}^{\underline{M}_i} = \tilde{X}^{\underline{M}_i}(\xi^i)$. Then, $\tilde{X}^{\underline{M}_i}(\xi^i)$ however can be absorbed into $X_0^{\underline{M}_i}(\xi^i)$, and hence $\tilde{X}^{\underline{M}_i}$ does not appear in the semiclassical action after all. For F-string case it has already been discussed in [20]. The vanishing of the longitudinal modes for the F-string is plausible also from the equivalence between the Nambu-Goto action and Polyakov action. In the Polyakov action the longitudinal modes should be canceled by conformal ghosts while the ghosts are not included in the Nambu-Goto action. But in the case of the Nambu-Goto this cancellation comes from the requirement that the static gauge should be maintained. Thus, anyway, it is sufficient to consider the transverse fluctuations.

We should comment on the scaling of brane tension. The common radius of AdS_5 and S^5, R, can be factored out to be an overall factor of the action. This may replace T_p by

$$T_p R^{p+1} = t_p \lambda^{\frac{p+1}{4}} \,, \quad t_p = \left(g_s (2\pi)^{\frac{p-1}{2}} \right)^{-1} \,, \quad \sqrt{\lambda} \equiv \frac{R^2}{2\pi\alpha'} \,. \tag{3.1}$$

We make $\lambda^{\frac{p+1}{4}}$ be absorbed into the fluctuations $\tilde{Z}$ as $\tilde{Z} \to \lambda^{-\frac{p+1}{8}}\tilde{Z}$, thus considering quadratic fluctuations is equivalent to considering the large λ limit. The cancellation of the classical contributions is necessary for the consistent large λ limit.[3]

We will see that the semiclassical AdS-brane actions completely agree with the non-relativistic AdS-brane actions derived in [19]. The bosonic and the fermionic fluctuations will be separately discussed below. First the bosonic ones are evaluated for each of the D-brane configurations. Then the fermionic ones are done in a unified way.

3.1 Bosonic fluctuations

From now on let us consider the bosonic fluctuations. Hereafter the world-sheet Wick rotation is assumed when we take a Euclidean brane with $s = 1$. The metric of $AdS_5 \times S^5$ we take is given in (A.1) and (A.2).

[3] Now one sees the direct correspondence between the semiclassical limit and the non-relativistic limit arranged in [19] (see equations (5.1) and (5.2) there).

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D-string (F-string). The bosonic part of the action is given by

$$\mathcal{L}_B = \sqrt{s\det(g_B + F)}\, d^2\xi\,. \tag{3.2}$$

Let us consider the AdS_2-brane (i.e., (2,0)-brane) and expand (3.2) around the classical solution

$$t = t(\xi^0)\,, \quad \rho = \rho(\xi^1)\,.$$

Here t and ρ depend only on ξ^0 and ξ^1, respectively. The induced metric is

$$g_{0ij} = -\cosh^2\rho\partial_i t\partial_j t + \partial_i\rho\partial_j\rho \ = \mathrm{diag}\,(-\cosh^2\rho(\partial_0 t)^2, (\partial_1\rho)^2)\,,$$

and the world-sheet geometry is AdS_2 rather than flat. It is an easy task to derive

$$\sqrt{s\det g_0} = \cosh\rho\partial_0 t\partial_1\rho\,, \qquad g_0^{ij} = \mathrm{diag}\left(-\frac{1}{\cosh^2\rho(\partial_0 t)^2}, \frac{1}{(\partial_1\rho)^2}\right).$$

Since the induced metric is expanded as follows:

$$\begin{aligned} g_{Bij} &= g_{0ij} + g_{2ij} + \cdots\,, \qquad g_{2ij} = \sum_{p=1}^{3}\sinh^2\rho\partial_i\tilde{\phi}_p\partial_j\tilde{\phi}_p + \partial_i\tilde{\gamma}\partial_j\tilde{\gamma} + \sum_{q=1}^{4}\partial_i\tilde{\varphi}_q\partial_j\tilde{\varphi}_q\,, \\ F &= d\tilde{A} \equiv F_1\,, \end{aligned}$$

the quadratic part of $\mathcal{L}_B$ is given by

$$\begin{aligned} \mathcal{L}_{2B} &= \frac{1}{2}\sqrt{s\det g_0}\,\left[g_0^{ij}g_{2ij} + \frac{1}{2}F_{1ij}F_1^{ij}\right]\,, \\ &= \frac{1}{2}\sqrt{s\det g_0}\,\left[g_0^{ij}(\sinh^2\rho\partial_i\tilde{\phi}_p\partial_j\tilde{\phi}_p + \partial_i\tilde{\gamma}\partial_j\tilde{\gamma} + \partial_i\tilde{\varphi}_q\partial_j\tilde{\varphi}_q) + \frac{1}{2}F_{1ij}F_1^{ij}\right]\,. \end{aligned} \tag{3.3}$$

By rescaling $\tilde{\phi}_p$ as $\bar{\phi}_p = \sinh\rho\tilde{\phi}_p$, the first term in (3.3) can be rewritten as

$$\frac{1}{2}\sqrt{s\det g_0}\,g_0^{ij}\sinh^2\rho\partial_i\tilde{\phi}_p\partial_j\tilde{\phi}_p = \frac{1}{2}\sqrt{s\det g_0}\,\left[g_0^{ij}\partial\bar{\phi}_p\partial\bar{\phi}_p + 2\bar{\phi}_p^2\right]\,,$$

where a partial integration has been performed. Thus one can obtain that

$$S_{2B} = t_1\int d^2\xi\sqrt{s\det g_0}\,\left[\frac{1}{2}g_0^{ij}(\partial_i\bar{\phi}_p\partial_j\bar{\phi}_p + \partial_i\tilde{\gamma}\partial_j\tilde{\gamma} + \partial_i\tilde{\varphi}_q\partial_j\tilde{\varphi}_q) + \bar{\phi}_p^2 + \frac{1}{4}F_{1ij}F_1^{ij}\right] \tag{3.4}$$

where R is absorbed into fluctuations. This is just the non-relativistic AdS D-string action derived in [19]. This reduces to the non-relativistic AdS F-string action derived in [20, 9] by setting $\tilde{A}$ to be zero.

D3-brane. The bosonic part of the D3-brane action is[4]

$$\mathcal{L}_B = \sqrt{s \det(g_B + F)}\, d^4\xi + C^{(4)} \ .$$

Let us consider the $AdS_3\times S^1$-brane (i.e., (3,1)-brane) first. The classical solution is given by

$$(t, \rho, \phi_1; \gamma) = (t(\xi^0), \rho(\xi^1), \phi_1(\xi^2); \gamma(\xi^3)) \ .$$

and the induced metric can be expanded around it as

$$\begin{aligned}
g_B &= g_0 + g_2 + \cdots \ , \\
g_{0ij} &= \mathrm{diag}(-\cosh^2\rho(\partial_0 t)^2, (\partial_1\rho)^2, \sinh^2\rho(\partial_2\phi_1)^2, (\partial_3\gamma)^2) \ , \\
g_{2ij} &= \sinh^2\rho\cos^2\phi_1 \sum_{p=2,3} \partial_i\tilde{\phi}_p\partial_j\tilde{\phi}_p + \cos^2\gamma\sum_{q=1}^{4}\partial_i\tilde{\varphi}_q\partial_j\tilde{\varphi}_q \, .
\end{aligned}$$

Defining the new angle variables:

$$\bar{\phi}_p \equiv \sinh\rho\cos\phi_1\tilde{\phi}_p \, , \qquad \bar{\varphi}_q \equiv \cos\gamma\tilde{\varphi}_q \, ,$$

the bosonic quadratic action of the DBI part can be rewritten as

$$\begin{aligned}
&\frac{1}{2}\sqrt{s\det(g_B+F)}\, g_0^{ij} g_{2ij} \\
&\quad = \sqrt{s\det g_0}\left[\frac{1}{2}g_0^{ij}(\partial_i\bar{\phi}_p\partial_j\bar{\phi}_p + \partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q) + \frac{1}{2}(3\bar{\phi}_p^2 - \bar{\varphi}_q^2) + \frac{1}{4}F_{1ij}F_1^{ij}\right] \, , \qquad (3.5)
\end{aligned}$$

where we have performed partial integrations. Then it is turn to consider the WZ part. The $C^{(4)}$ is now expressed as

$$C^{(4)} = 4\sqrt{s}i(-\cosh\rho\sinh^3\rho\cos^2\phi_1 dt d\rho d\phi_1\phi_2 d\phi_3 + \cos^4\gamma d\gamma\varphi_1 d\varphi_2\cdots d\varphi_4) \, ,$$

up to an exact term, and it can be expanded in terms of the fluctuations as

$$C^{(4)} = C_2^{(4)} + \cdots \, ,$$

and the integration of the quadratic part is given by

$$\begin{aligned}
\int C_2^{(4)} &= -4\sqrt{s}i\int \cosh\rho\sinh^3\rho\cos^2\phi_1 dt d\rho d\phi_1\tilde{\phi}_2 d\tilde{\phi}_3 \\
&= -4\sqrt{s}i\int d^4\xi\ \cosh\rho\sinh\rho\partial_0 t\partial_1\rho\partial_2\phi_1\bar{\phi}_2\partial_3\bar{\phi}_3 \\
&= -4\sqrt{s}i\int_\Sigma \mathrm{vol}_{\Sigma_3}\,\bar{\phi}_2 d\bar{\phi}_3 \, , \qquad (3.6)
\end{aligned}$$

where Σ_3 is the worldvolume extending in the AdS_5. Combining (3.5) and (3.6) , one can obtain the following quadratic action

$$\begin{aligned}
S_{2B} &= t_3\int d^4\xi\,\sqrt{s\det g_0}\left[\frac{1}{2}g_0^{ij}(\partial_i\bar{\phi}_p\partial_j\bar{\phi}_p + \partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q) + \frac{1}{2}(3\bar{\phi}_p^2 - \bar{\varphi}_q^2) + \frac{1}{4}F_{1ij}F_1^{ij}\right] \\
&\quad -4\sqrt{s}it_3\int_\Sigma \mathrm{vol}_{\Sigma_3}\bar{\phi}_2 d\bar{\phi}_3 \ . \qquad (3.7)
\end{aligned}$$

[4] The supersymmetric actions of D3-branes on the $AdS_5\times S^5$ and the pp-wave are constructed in [26] and [27], respectively.

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This is nothing but the non-relativistic AdS D3-brane action derived in [19].

On the other hand, we may consider (1,3)-brane. The shape is actually $R\times\mathrm{S}^3$ and so this is related to the giant graviton [28] rather than AdS-branes. The static solution is given by

$$(t;\gamma,\varphi_1,\varphi_2)=(t(\xi^0);\gamma(\xi^1),\varphi_1(\xi^2),\varphi_2(\xi^3))$$

as it corresponds to a giant graviton. By carrying out the same analysis, we can obtain the following quadratic action:

$$\begin{aligned}S_{2B}&=t_3\int\!d^4\xi\,\sqrt{s\det g_0}\,\left[\frac{1}{2}g_0^{ij}(\partial_i\tilde{\rho}\partial_j\tilde{\rho}+\partial_i\bar{\phi}_p\partial_j\bar{\phi}_p+\partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q)\right.\\&\qquad\left.+\frac{1}{2}(\tilde{\rho}^2+\bar{\phi}_p^2-3\bar{\varphi}_q^2)+\frac{1}{4}F_{1ij}F_1^{ij}\right]+4\sqrt{s}it_3\int_\Sigma\mathrm{vol}_{\Sigma_3'}\,\bar{\varphi}_3d\bar{\varphi}_4\end{aligned}\tag{3.8}$$

where $p=1,2,3$ and $q=3,4$. Σ_3' is the worldvolume extending in the S^5.

D5-brane. The bosonic part of the D5-brane action is

$$\mathcal{L}_B=\sqrt{s\det(g_B+F)}\,d^6\xi+C^{(6)}\,.$$

Let us consider the $\mathrm{AdS}_4\times\mathrm{S}^2$-brane (i.e., (4,2)-brane) first. The classical solution is

$$(t,\rho,\phi_1,\phi_2;\gamma,\varphi_1)=(t(\xi^0),\rho(\xi^1),\phi_1(\xi^2),\phi_2(\xi^3);\gamma(\xi^4),\varphi_1(\xi^5))\,.$$

The induced metric is expanded as

$$g_B=g_0+g_2+\cdots\,,$$

and the zeroth and the second order parts are given by, respectively,

$$\begin{aligned}g_{0ij}&=\mathrm{diag}(-\cosh^2\rho(\partial_0t)^2,(\partial_1\rho)^2,\sinh^2\rho(\partial_2\phi_1)^2,\sinh^2\rho\cos^2\phi_1(\partial_3\phi_2)^2,\\&\qquad(\partial_4\gamma)^2,\cos^2\gamma(\partial_5\varphi_1)^2)\,,\\g_{2ij}&=\sinh^2\rho\cos^2\phi_1\cos^2\phi_2\partial_i\tilde{\phi}_3\partial_j\tilde{\phi}_3+\sum_{q=2}^{4}\cos^2\gamma\cos^2\varphi_1\partial_i\tilde{\varphi}_q\partial_j\tilde{\varphi}_q\,.\end{aligned}$$

Defining the new variables

$$\bar{\phi}_3\equiv\sinh\rho\cos\phi_1\cos\phi_2\tilde{\phi}_3\,,\quad\bar{\varphi}_q\equiv\cos\gamma\cos\varphi_1\tilde{\varphi}_q\,,$$

one can rewrite the quadratic part of the bosonic DBI action as

$$\frac{1}{2}\sqrt{s\det(g_B+F)}\,g_0^{ij}g_{2ij}=\sqrt{s\det g_0}\,\left[\frac{1}{2}g_0^{ij}(\partial_i\bar{\phi}_3\partial_j\bar{\phi}_3+\partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q)+2\bar{\phi}_3^2-\bar{\varphi}_q^2+\frac{1}{4}F_{1ij}F_1^{ij}\right]\,,\tag{3.9}$$

where we have performed partial integrations. Then let us consider the WZ part. Since in the present metric we have

$$dC^{(6)}=h_7|_{\mathrm{bosonic}}=\frac{\sqrt{s}i}{3!5}(\epsilon_{a_1\cdots a_5}e^{a_1}\cdots e^{a_5}-\epsilon_{a_1'\cdots a_5'}e^{a_1'}\cdots e^{a_5'})F\,,$$

the $C^{(6)}$ is expanded as

$$C^{(6)} = C_2^{(6)} + \cdots .$$

The integration of $C_2^{(6)}$ is given by

$$\begin{aligned}\int C_2^{(6)} &= 4\sqrt{s}i \int \cosh\rho \sinh^3\rho \cos^2\phi_1 \cos\phi_2 dt d\rho d\phi_1 d\phi_2 \tilde{\phi}_3 \ F_1 \\ &= 4\sqrt{s}i \int_\Sigma \mathrm{vol}_{\Sigma_4}\, \bar{\phi}_3 F_1 \,, \end{aligned} \tag{3.10}$$

where $\mathrm{vol}_{\Sigma_4} = \cosh\rho\sinh^2\rho\cos\phi_1 dt d\rho d\phi_1 d\phi_2$. Combining (3.9) and (3.10), one can find the quadratic action

$$\begin{aligned} S_{2B} &= t_5 \int d^6\xi \sqrt{s\det g_0}\left[\frac{1}{2}g_0^{ij}(\partial_i\bar{\phi}_3\partial_j\bar{\phi}_3 + \partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q) + 2\bar{\phi}_3^2 - \bar{\varphi}_q^2 + \frac{1}{4}F_{1ij}F_1^{ij}\right] \\ &\quad + 4\sqrt{s}it_5 \int_\Sigma \mathrm{vol}_{\Sigma_4}\, \bar{\phi}_3 F_1 \,. \end{aligned} \tag{3.11}$$

This is nothing but the non-relativistic AdS D5-brane action derived in [19].

It is also interesting to consider the $\mathrm{AdS}_2\times\mathrm{S}^4$-brane case.[5] The classical solution is given by

$$(t,\rho;\gamma,\varphi_1,\varphi_2,\varphi_3) = (t(\xi^0),\rho(\xi^0);\gamma(\xi^0),\varphi_1(\xi^0),\varphi_2(\xi^0),\varphi_3(\xi^0))\,.$$

By following the same line, we can obtain the following quadratic action

$$\begin{aligned} S_{2B} &= t_5 \int d^6\xi \sqrt{s\det g_0}\left[\frac{1}{2}g_0^{ij}(\partial_i\bar{\phi}_p\partial_j\bar{\phi}_p + \partial_i\bar{\varphi}_4\partial_j\bar{\varphi}_4) + \bar{\phi}_p^2 - 2\bar{\varphi}_4^2 + \frac{1}{4}F_{1ij}F_1^{ij}\right] \\ &\quad - 4\sqrt{s}it_5 \int_\Sigma \mathrm{vol}_{\Sigma_4'}\, \bar{\varphi}_4 F_1 \end{aligned} \tag{3.12}$$

where $p = 1,2,3$. This also agrees with the result of [19].

D7-brane. The bosonic part of the D5-brane action is

$$\mathcal{L}_B = \sqrt{s\det(g_B + F)}\, d^8\xi + C^{(8)}\,.$$

Let us consider the $\mathrm{AdS}_5\times\mathrm{S}^3$-brane (i.e., (5,3)-brane) classical solution

$$(t,\rho,\phi_1,\phi_2,\phi_3;\gamma,\varphi_1,\varphi_2) = (t(\xi^0),\rho(\xi^1),\phi_1(\xi^2),\phi_2(\xi^3),\phi_3(\xi^4);\gamma(\xi^5),\varphi_1(\xi^6),\varphi_2(\xi^7))\,.$$

The induced metric is expanded as

$$g_B = g_0 + g_2 + \cdots ,$$

and the zeroth and the second order parts are given by, respectively,

$$\begin{aligned} g_{0ij} &= \mathrm{diag}(-\cosh^2\rho(\partial_0 t)^2, (\partial_1\rho)^2, \sinh^2\rho(\partial_2\phi_1)^2, \sinh^2\rho\cos^2\phi_1(\partial_3\phi_2)^2, \\ &\qquad \sinh^2\rho\cos^2\phi_1\cos^2\phi_2(\partial_4\phi_3)^2, (\partial_5\gamma)^2, \cos^2\gamma(\partial_6\varphi_1)^2, \cos^2\gamma\cos^2\varphi_1(\partial_7\varphi_2)^2)\,, \\ g_{2ij} &= \sum_{q=3,4}\cos^2\gamma\cos^2\varphi_1\cos^2\varphi_2\partial_i\tilde{\varphi}_q\partial_j\tilde{\varphi}_q\,. \end{aligned}$$

[5]This shape may be related to a giant Wilson loop [29].

Defining the new variable, $\bar{\varphi}_q \equiv \cos\gamma\cos\varphi_1\cos\varphi_2\tilde{\varphi}_q$, one can rewrite the quadratic part of the bosonic DBI part as

$$\frac{1}{2}\sqrt{s\det(g_B+F)}\,g_0^{ij}g_{2ij} = \sqrt{s\det g_0}\,\left[\frac{1}{2}g_0^{ij}\partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q - \frac{3}{2}\bar{\varphi}_q^2 + \frac{1}{4}F_{1ij}F_1^{ij}\right] ,$$

where we have performed partial integrations.

Then let us consider the WZ part. We can start from the following expression,

$$dC^{(8)} = h_9|_{\text{bosonic}} = \frac{\sqrt{s}i}{2\cdot 3!\cdot 5}\left(\epsilon_{a_1\cdots a_5}e^{a_1}\cdots e^{a_5} - \epsilon_{a'_1\cdots a'_5}e^{a'_1}\cdots e^{a'_5}\right)F^2 ,$$

and $C^{(8)}$ can be expanded as

$$C^{(8)} = C_2^{(8)} + \cdots .$$

The integration of the quadratic part is given by

$$\int C_2^{(8)} = -2\sqrt{s}i\int_\Sigma \text{vol}_{\Sigma_5}\tilde{A}F_1 ,$$
$$\text{vol}_{\Sigma_5} = \cosh\rho\sinh^3\rho\cos^2\phi_1\cos\phi_2 dtd\rho d\phi_1 d\phi_2 d\phi_3 .$$

Combining the results we find the fluctuation action

$$S_{2B} = t_7\int d^8\xi\,\sqrt{s\det g_0}\,\left[\frac{1}{2}g_0^{ij}\partial_i\bar{\varphi}_q\partial_j\bar{\varphi}_q - \frac{3}{2}\bar{\varphi}_q^2 + \frac{1}{4}F_{1ij}F_1^{ij}\right] - 2\sqrt{s}it_7\int_\Sigma \text{vol}_{\Sigma_5}\,\tilde{A}F_1 . \quad (3.13)$$

This is nothing but the non-relativistic AdS D7-brane action derived in [19].

Next let us consider the $AdS_3\times S^5$-brane case, which is slightly different from the $AdS_5\times S^3$-brane case. For $AdS_3\times S^5$-brane case, the classical solution is given by

$$(t,\rho,\phi_1;\gamma,\varphi_1,\varphi_2,\varphi_3,\varphi_4) = (t(\xi^0),\rho(\xi^1),\phi_1(\xi^2);\gamma(\xi^3),\varphi_1(\xi^4),\varphi_2(\xi^5),\varphi_3(\xi^6),\varphi_4(\xi^7)) ,$$

and we can derive the following action:

$$S_{2B} = t_7\int d^8\xi\,\sqrt{s\det g_0}\,\left[\frac{1}{2}g_0^{ij}\partial_i\bar{\phi}_p\partial_j\bar{\phi}_p + \frac{3}{2}\bar{\phi}_p^2 + \frac{1}{4}F_{1ij}F_1^{ij}\right] + 2\sqrt{s}it_7\int_\Sigma \text{vol}_{\Sigma'_5}\,\tilde{A}F_1 , \quad (3.14)$$

where $p = 2, 3$. This is also nothing but the non-relativistic AdS D7-brane action derived in [19]. Here it should be noted that no tachyonic mode is contained in the mass spectrum. This fact would basically be based on the topological reason that the S^5 part of the brane is wrapped around the S^5 part of the $AdS_5\times S^5$ background. It should stabilize the solution even if the supersymmetries are broken due to some effect.

3.2 Fermionic fluctuations

Next we shall consider the fermionic fluctuations. As mentioned before, the fermionic part can be discussed in a unified way in comparison to the bosonic part, i.e., it does not almost depend on the dimensionality of Dp-branes. Hence we will discuss the fermionic part of all D-branes at a time.

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Expanding about the classical solution, we can derive the quadratic action for the fermionic variables:

$$
\begin{aligned}
S_{2F} &= T_p \int d^{p+1}\xi \, \sqrt{s \det g_0} \, g_0^{ij} i\bar{\tilde{\theta}}\gamma_i (D_j\tilde{\theta})_0 \\
&\quad - T_p \int_\Sigma \frac{\sqrt{s}}{p!} e_0^{A_1} \cdots e_0^{A_p} \bar{\tilde{\theta}} \Gamma_{A_1\cdots A_p} (\sigma)^{\frac{p+1}{2}} i\sigma_2 (D\tilde{\theta})_0 \,,
\end{aligned}
$$

where the covariant derivative is defined as

$$
(D\tilde{\theta})_0 = d\tilde{\theta} + \frac{1}{2} e_0^A \widehat{\Gamma}_A i\sigma_2 \tilde{\theta} + \frac{1}{4}\omega_0^{AB}\Gamma_{AB}\tilde{\theta} \,, \qquad \gamma_i = (e_0)_i^A \Gamma_A
$$

where subscript 0 means classical value. The quadratic action $S_{2B} + S_{2F}$ is invariant under the following fermionic symmetry

$$
\begin{aligned}
\delta_\kappa \tilde{\theta} &= (1 + \Gamma_0)\kappa \,, \\
\Gamma_0 &= \frac{s\sqrt{-s}}{\sqrt{s \det g_0}} (\sigma)^{n - \frac{p-3}{2}} i\sigma_2 \frac{1}{(p+1)!} \epsilon^{i_1\cdots i_{p+1}} (e_0)_{i_1}^{A_0} \cdots (e_0)_{i_{p+1}}^{A_p} \Gamma_{A_0\cdots A_p} \,, \\
&= \sqrt{-s}\Gamma^{A_0\cdots A_p}\rho \equiv M \,, \quad \rho = \begin{cases} \sigma_1 & \text{for } p = 1 \bmod 4 \\ i\sigma_2 & \text{for } p = 3 \bmod 4 \end{cases} .
\end{aligned} \tag{3.15}
$$

For F-string case, $\rho = \sigma_3$. This symmetry is inherited from the κ-symmetry of Dp-brane action

$$
\begin{aligned}
\delta_\kappa \theta &= (1 + \Gamma)\kappa \,, \\
\Gamma &= \frac{s\sqrt{-s}}{\sqrt{s \det(g + \mathcal{F})}} \sum_{n=0} \frac{1}{2^n n!} \Gamma^{j_1k_1\cdots j_nk_n} \mathcal{F}_{j_1k_1} \cdots \mathcal{F}_{j_nk_n} \\
&\quad \times (-1)^n (\sigma)^{n - \frac{p-3}{2}} i\sigma_2 \frac{1}{(p+1)!} \epsilon^{i_1\cdots i_{p+1}} \Gamma_{i_1\cdots i_{p+1}} \,,
\end{aligned}
$$

where $\Gamma_i = \mathbf{L}_i^A \Gamma_A$. Fixing the fermionic symmetry (3.15) with the condition

$$
\tilde{\theta}_+ = 0 \,, \quad \tilde{\theta}_\pm = P_\pm \tilde{\theta}_\pm \,, \quad P_\pm = \frac{1}{2}(1 \pm M) \,,
$$

we obtain the gauge fixed action:

$$
S_{2F} = t_p \int d^{p+1}\xi \, \sqrt{s \det g_0} \; 2i g_0^{ij} \bar{\vartheta} \gamma_i (D_j \vartheta)_0 \,,
$$

where we have absorbed R by rescaling $\tilde{\theta}$ and denoted $\vartheta \equiv \tilde{\theta}_-$. This is the fermionic part of non-relativistic AdS Dp-brane action derived in [19].

Finally we comment on the mass term contained in $(D\vartheta)_0$. The vielbein and spin connection on the $(p+1)$-dimensional world-volume for our classical solutions are given by

$$
\hat{e}_i^\alpha = \partial_i X_0^M (e_0)_M^\alpha \,, \quad \hat{\omega}_i^{\alpha\beta} = \partial_i X_0^M (\omega_0)_M^{\alpha\beta} \,,
$$

where α is the tangential-vector index. By using the $(p+1)$-dimensional spinorial derivative defined by

$$
\nabla_i = \partial_i + \frac{1}{4} \hat{\omega}_i^{\alpha\beta} \Gamma_{\alpha\beta} \,,
$$

one can find that

$$(D_i\vartheta)_0 = \nabla_i\vartheta + \frac{1}{2}(e_0)_i^A\widehat{\Gamma}_A i\sigma_2\vartheta\,.$$

So for (m, n)-branes

$$g_0^{ij}\bar{\vartheta}\gamma_i\frac{1}{2}(e_0)_j^A\widehat{\Gamma}_A i\sigma_2\vartheta = -\frac{m+n}{2}g_0^{ij}\bar{\vartheta}\mathcal{I}i\sigma_2\vartheta\,,$$

where we have used $\mathcal{I}\theta = -\mathcal{J}\theta$ which follows from $\Gamma_{01\cdots 9}\theta = \theta$. Thus

$$S_{2F} = t_p\int d^{p+1}\xi\,\sqrt{s\det g_0}\;2i\left[g_0^{ij}\bar{\vartheta}\gamma_i\nabla_j\vartheta - \frac{m+n}{2}\bar{\vartheta}\mathcal{I}i\sigma_2\vartheta\right]\,. \tag{3.16}$$

This is the action for 8 massive fermions propagating on $\mathrm{AdS}_m\times\mathrm{S}^n$ for the (m, n)-branes.

Thus we have shown that the non-relativistic actions constructed in [19] can be reproduced from the semiclassical limit around static 1/2 BPS D-brane configurations.

4. Gauge theory side

In the previous section we have shown that the non-relativistic limit is nothing but a semiclassical approximation around a static configuration. In this section, we argue the corresponding composite operators in the gauge theory side, according to the semiclassical interpretation for the non-relativistic limit.

4.1 BMN dictionary

Before considering the non-relativistic limit, it is helpful to remember the pp-wave case [5, 17]. The pp-wave string can be seen as a semiclassical approximation around a BPS particle solution rotating at the velocity of light. The solution has a U(1) charge J associated with the rotation. Then in the gauge theory side it corresponds to a single trace operator

$$\mathrm{Tr}(Z^J)\,, \qquad Z \equiv \phi_5 + i\phi_6\,. \tag{4.1}$$

The solution is invariant under SO(4)×SO(4) and the fluctuations are represented by insertions of impurities:

$$D_\mu Z \quad (\mu = 0, 1, 2, 3)\,, \qquad \phi_I \quad (I = 1, 2, 3, 4)\,,$$

into the vacuum operator (4.1). Here it should be noted that $D_\mu Z$ has bare scaling dimension 2 but Z carries the U(1) charge 1, and hence $\Delta - J = 1$.

4.2 Dictionary in non-relativistic limit

Let us consider the non-relativistic string theory. The world-sheet geometry is AdS_2 and it ends on the AdS boundary. Hence a straight Wilson line[6]

$$W = \mathrm{Tr}\;\mathrm{P}\exp\left[\int dt\;(iA_0 + \phi)\right]\,, \qquad \phi = \sum_{i=1}^{6} n^i\phi_i\,.$$

[6]Here we discuss a single AdS_2-brane and the representation of the corresponding Wilson line is represented by a single box in terms of Young tableau.

is contained in the gauge theory side. This is 1/2 BPS and invariant under SO(3)×SO(5). Hereafter we will fix the six-dimensional vector n^i as $n^i = (0, \ldots, 0, 1)$.

In analogy with the semiclassical interpretation for the Penrose limit [17], it is plausible to identify the Wilson line with the vacuum operator. From the mass spectrum of the non-relativistic string, its transverse symmetry is SO(3)×SO(5) and it should be related to the fluctuations. Then we argue that the fluctuations would be represented by the insertions of the following impurities:

$$D_a\phi_6 + iF_{a0} \quad (a = 1, 2, 3)\,, \qquad \phi^{a'} \quad (a' = 1, \ldots, 5)\,. \tag{4.2}$$

For the AdS directions, the field strength is included in comparison to the BMN case. Here we should note the difference of the scaling dimensions 1 between AdS$_5$ and S^5 directions. It is closly related to the difference between the masses of non-relativistic string as we will see later.

We will see some supports for the dictionary (4.2) below.

4.3 Wilson loop expansion

The dictionary (4.2) is supported also by another argument based on expanding a Wilson loop, following the method developed in [30].

Let us consider the Taylor expansion of Wilson line

$$W(C) = \mathrm{Tr}\left[\mathrm{P}\ \exp\left(\int_{s_i}^{s_f} ds\ \left(iA_\mu(x(s))\dot{x}^\mu(s) + \phi_i \dot{y}^i(s)\right)\right)\right]$$

around the straight Wilson line C_0

$$x^0_{C_0}(s) = s\,, \qquad \dot{y}^i_{C_0}(s) = (0, 0, 0, 0, 0, 1)\,,$$

by considering a small deformation like $W(C) = W(C_0 + \delta C)$ [30].

The Wilson line can be expanded as

$$\begin{aligned} W(C) &= W(C_0) + \int_{s_i}^{s_f} ds\, \delta x^\mu(s)\ \frac{\delta W(C)}{\delta x^\mu(s)}\bigg|_{C=C_0} + \int_{s_i}^{s_f} ds\, \delta\dot{y}^i(s)\ \frac{\delta W(C)}{\delta \dot{y}^i(s)}\bigg|_{C=C_0} \\ &\quad + \frac{1}{2}\int_{s_i}^{s_f} ds_1 \int_{s_i}^{s_f} ds_2\ \delta x^\mu(s_1)\delta x^\nu(s_2) \frac{\delta^2 W(C)}{\delta x^{\mu(s_1)}\delta x^\nu(s_2)}\bigg|_{C=C_0} + \cdots\,. \end{aligned} \tag{4.3}$$

The zeroth order term is nothing but the straight Wilson line. Then the first order term is evaluated as

$$\begin{aligned} \frac{\delta W(C)}{\delta x^\mu(s)}\bigg|_{C=C_0} &= \mathrm{Tr}\bigg[\left(iF_{\mu\nu}(x(s))\dot{x}^\nu(s) + D_\mu\phi_i(x(s))\dot{y}^i(s)\right) \times \\ &\qquad \times \mathrm{P}\ \exp\left(\int_{s+s_i}^{s+s_f} ds'\ \left(iA_\mu(x(s'))\dot{x}^\mu(s') + \phi_i\dot{y}^i(s')\right)\right)\bigg]\bigg|_{C=C_0} \\ &= \mathrm{Tr}\left[(iF_{\mu 0} + D_\mu\phi_6)\,\mathrm{P}\ \exp\left(\int_{-\infty}^{\infty} dt\ (iA_0 + \phi_6)\right)\right]\,, \end{aligned} \tag{4.4}$$

$$\frac{\delta W(C)}{\delta \dot{y}^i(s)}\bigg|_{C=C_0} = \mathrm{Tr}\left[\phi_i\ \mathrm{P}\ \exp\left(\int_{-\infty}^{\infty} dt\ (iA_0 + \phi_6)\right)\right]\,. \tag{4.5}$$

The fluctuations are expanded as

$$\delta x^\mu(s) = \sum_{n=-\infty}^{\infty} \delta x_n^\mu \mathrm{e}^{2\pi ins/(s_f-s_i)}\,, \qquad \delta\dot{y}^i(s) = \sum_{n=-\infty}^{\infty} \delta\dot{y}_n^i \mathrm{e}^{2\pi ins/(s_f-s_i)}\,,$$

and those give the following relations:

$$\sum_n \delta x_n^\mu \int_{s_i}^{s_f} ds\, \left.\frac{\delta W(C)}{\delta x^\mu(s)}\right|_{C=C_0} \mathrm{e}^{2\pi ins/(s_f-s_i)} = \delta x_0^\mu \times \; (4.4)\,,$$
$$\sum_n \delta\dot{y}_n^i \int_{s_i}^{s_f} ds\, \delta\dot{y}^i(s) \left.\frac{\delta W(C)}{\delta\dot{y}^i(s)}\right|_{C=C_0} \mathrm{e}^{2\pi ins/(s_f-s_i)} = \delta\dot{y}_0^i \times \; (4.5)\,.$$

By letting

$$\delta x^0 = \delta\dot{y}^6 = 0\,, \tag{4.6}$$

we are left with the impurities given in (4.2). The condition (4.6) will be supported from a supersymmetry argument.

Supersymmetry. The straight Wilson line $W(C_0)$ is 1/2 BPS. As one can easily see, the supersymmetry variation[7] $\delta_\epsilon W(C_0)$ vanishes when

$$(\dot{x}^\mu_{C_0}\Gamma_\mu - i\dot{y}^i_{C_0}\Gamma_i)\epsilon = 0\,,$$

which implies the locally supersymmetric condition

$$(\dot{x}^\mu_{C_0})^2 - (\dot{y}^i_{C_0})^2 = 0\,.$$

The straight Wilson line $W(C_0)$ indeed satisfies the condition.

Then let us consider linear fluctuations around $W(C_0)$. The supersymmetry transformation for the fluctuations vanishes when the following relation is satisfied:

$$\left[(\dot{x}^\mu_{C_0} + \delta\dot{x}^\mu)\Gamma_\mu - i(\dot{y}^i_{C_0} + \delta\dot{y}^i)\Gamma_i\right]\epsilon = 0\,.$$

It implies that

$$\delta\dot{x}^0 - \delta\dot{y}^6 = 0\,.$$

On the other hand, one may choose as

$$\delta\dot{x}^0 + \delta\dot{y}^6 = 0$$

by using the SO(1,1) symmetry. Thus the linear fluctuations are locally supersymmetric and 1/2 BPS when $\delta\dot{x}^0 = 0$ and $\delta\dot{y}^6 = 0$. The former means $\delta x^0 = 0$ as the Wilson line is characterized by x^μ. In fact, we have just seen above that impurities (4.2) appeared as fluctuations satisfying $\delta x^0 = \delta\dot{y}^6 = 0$.

[7] The supersymmetry transformation is given by $\delta_\epsilon A_\mu = i\bar{\Psi}\Gamma_\mu\epsilon$ and $\delta_\epsilon\phi_i = i\bar{\Psi}\Gamma_i\epsilon$.

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4.4 Supergravity modes in non-relativistic limit

Next let us consider the fluctuations by focusing upon the supergravity modes. These should be BPS and protected from quantum corrections. Then the mass dimensions of the fluctuations can be computed following [2].

We show that the mass dimensions of the fluctuations propagating in AdS_2 evaluated at the boundary are equal to the conformal dimensions of the conjectured impurities. The masses of the fluctuations are easily derived from (3.4) with $F = 0$ and (3.16) as

$$m_B^2 = 2 \ \text{ for } \ \bar{\phi}\,, \quad m_B^2 = 0 \ \text{ for } \ \tilde{\gamma},\ \tilde{\varphi} \tag{4.7}$$

and $m_F^2 = 1$ for ϑ. The mass dimensions of scalars can be evaluated at the boudary as follows [2]:

$$\Delta = \frac{1}{2}\Big(1 + \sqrt{1 + 4m^2}\Big)\,. \tag{4.8}$$

By substituting (4.7) for (4.8), it follows that

$$\Delta(\bar{\phi}) = 2 \quad \text{and} \quad \Delta(\tilde{\gamma}, \tilde{\varphi}) = 1\,.$$

On the other hand, the conformal dimensions of impurities are

$$2 \ \text{ for } \ D_a\phi_6 + iF_{a0} \quad \text{and} \quad 1 \ \text{ for } \ \phi^{a'}\,.$$

Thus we find the agreement of the mass dimensions of the fluctuations and the conformal dimensions of the impurities. This provides a further support of our conjecture.

Finally we comment on the stringy excitation modes. The quantum spectrum of the non-relativistic string has not been obtained yet by solving the theory. The world-sheet theory is free but the world-sheet geometry is AdS_2 rather than flat, and the quantization of it is more involved. Furthermore, unlike the pp-wave string case, the U(1) charge J is not included in the present case. Thus the BMN trick does not work and hence perturbation theory would be useless in comparing the gauge side with the string result. Anyway, it would be difficult to check the above dictionary at stringy level.

5. Summary and discussions

We have derived non-relativistic actions of string and D-branes on $AdS_5 \times S^5$ from a semi-classical approximation around static configurations. Then the AdS/CFT dictionary in the non-relativistic limit, which is a new solvable sector pointed out in [9], has been argued on the basis of the semiclassical interpretation and the symmetry argument. We speculate that a state of non-relativistic string would correspond to a small deformation of 1/2 BPS straight Wilson line.

Here it may be valuable to comment on the difference of setups between ours and [31, 32]. In the case of [31, 32] a deformed Wilson loop with "long" composite operators is considered and a large $U(1)_R$ charge is included as the length of the long operators unlike our case. Therefore perturbation theory works in the case of [31, 32] to check the correspondence even at stringy level and it is also an interesting direction to study the deformed Wilson loops furthermore (For the works in this direction, see [31 − 34]). On the other hand, it would be quite difficult in our case to test at the strigy level.

It would be another direction to study a semiclassical approximation of M-branes. The non-relativistic M-brane actions have already been obtained in our previous paper [19]. We will report on this issue in another place in the near future [35].

We hope that our result could be a new clue of the study of AdS/CFT correspondence.

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A. The supervielbeins on $\mathbf{AdS}_5\times\mathbf{S}^5$

Here we will construct the supervielbeins on the $\mathrm{AdS}_5\times\mathrm{S}^5$ background, in order to make the manuscript self-contained and fix the notations and conventions.

First of all, let us introduce the metric of $\mathrm{AdS}_5\times\mathrm{S}^5$ background:

$$ds^2_{AdS} = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho\Big[d\phi_1^2 + \cos^2\phi_1(d\phi_2^2 + \cos^2\phi_2 d\phi_3^2)\Big]\,, \tag{A.1}$$

$$ds^2_{S} = d\gamma^2 + \cos^2\gamma\Big[d\varphi_1^2 + \cos^2\varphi_1(d\varphi_2^2 + \cos^2\varphi_2\big(d\varphi_3^2 + \cos^2\varphi_3 d\varphi_4^2)\big)\Big]\,. \tag{A.2}$$

As we factor out the radii R of AdS_5 and S^5 to be an overall factor in the action, the geometrical objects collected here are for $R = 1$. Then for the AdS_5 part, one can read off from (A.1) the vielbein e^a $(a = 0, 1, \cdots, 4)$ as

$$e^a = (\cosh\rho dt,\, d\rho,\, \sinh\rho d\phi_1,\, \sinh\rho\cos\phi_1 d\phi_2,\, \sinh\rho\cos\phi_1\cos\phi_2 d\phi_3)\,.$$

The spin connection is defined by $de^a = -\omega^a{}_b e^b$, and the non-trivial components of the spin connection for the AdS_5 part, are

$$\begin{aligned}
&\omega^0{}_1 = \sinh\rho dt\,, \quad \omega^2{}_1 = \cosh\rho d\phi_1\,, \quad \omega^3{}_1 = \cosh\rho\cos\phi_1 d\phi_1\,, \quad \omega^3{}_2 = -\sin\phi_1 d\phi_2\,,\\
&\omega^4{}_1 = \cosh\rho\cos\phi_1\cos\phi_2 d\phi_3\,, \qquad \omega^4{}_2 = -\sin\phi_1\cos\phi_2 d\phi_3\,, \quad \omega^4{}_3 = -\sin\phi_2 d\phi_3\,.
\end{aligned}$$

Next, for the S^5 part, the vielbein $e^{a'}$ $(a' = 5, 6, \cdots, 9)$ is seen from (A.2) as

$$\begin{aligned}
e^{a'} = (&d\gamma,\, \cos\gamma d\varphi_1,\, \cos\gamma\cos\varphi_1 d\varphi_2,\\
&\cos\gamma\cos\varphi_1\cos\varphi_2 d\varphi_3,\, \cos\gamma\cos\varphi_1\cos\varphi_2\cos\varphi_3 d\varphi_4)\,.
\end{aligned}$$

The non-zero components of the spin connection are given by

$$
\begin{aligned}
&\omega^{6}{}_{5} = -\sin\gamma d\varphi_1\,, && \omega^{7}{}_{5} = -\sin\gamma\cos\varphi_1 d\varphi_2\,, && \omega^{7}{}_{6} = -\sin\varphi_1 d\varphi_2\,,\\
&\omega^{8}{}_{5} = -\sin\gamma\cos\varphi_1\cos\varphi_2 d\varphi_3\,, && \omega^{8}{}_{6} = -\sin\varphi_1\cos\varphi_2 d\varphi_3\,, && \omega^{8}{}_{7} = -\sin\varphi_2 d\varphi_3\,,\\
&\omega^{9}{}_{5} = -\sin\gamma\cos\varphi_1\cos\varphi_2\cos\varphi_3 d\varphi_4, && \omega^{9}{}_{6} = -\sin\varphi_1\cos\varphi_2\cos\varphi_3 d\varphi_4\,, &&\\
&\omega^{9}{}_{7} = -\sin\varphi_2\cos\varphi_3 d\varphi_4, && \omega^{9}{}_{7} = -\sin\varphi_3 d\varphi_4\,. &&
\end{aligned}
$$

Then it is turn to consider the supervielbeins on $\mathrm{AdS}_5\times\mathrm{S}^5$, which can be obtained via the coset construction with the coset superspace:

$$
\mathrm{AdS}_5\times\mathrm{S}^5 \sim \frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}\,. \tag{A.3}
$$

The super-$\mathrm{AdS}_5\times\mathrm{S}^5$ algebra is represented by the super Lie algebra $psu(2,2|4)$, whose commutation relations are given by

$$
\begin{aligned}
[P_a,P_b] &= \alpha^2 J_{ab}\,, & [P_{a'},P_{b'}] &= -\alpha^2 J_{a'b'}\,,\\
[P_a,J_{bc}] &= \eta_{ab}P_c-\eta_{ac}P_b\,, & [P_{a'},J_{b'c'}] &= \eta_{a'b'}P_{c'}-\eta_{a'c'}P_{b'}\,,\\
[J_{ab},J_{cd}] &= \eta_{bc}J_{ad}+\text{3-terms}\,, & [J_{a'b'},J_{c'd'}] &= \eta_{b'c'}J_{a'd'}+\text{3-terms}\,,\\
[Q_I,P_A] &= -\frac{\alpha}{2}Q_J(i\sigma_2)_{JI}\widehat{\Gamma}_A\,, & [Q_I,J_{AB}] &= -\frac{1}{2}Q_I\Gamma_{AB}\,,\\
\{Q_I,Q_J\} &= 2i\mathcal{C}\Gamma^A\delta_{IJ}h_+P_A - i\alpha\mathcal{C}\widehat{\Gamma}^{AB}(i\sigma_2)_{IJ}h_+J_{AB}\,, &&
\end{aligned} \tag{A.4}
$$

where $A=(a,a')$. $\alpha\equiv 1/R$ is set to be 1 in the body of the paper.

Then the gamma matrix $\Gamma^A\in$ Spin(1,9) satisfies

$$
\{\Gamma^A,\Gamma^B\}=2\eta^{AB}\,,\quad (\Gamma^A)^T=-\mathcal{C}\Gamma^A\mathcal{C}^{-1}\,,\quad \mathcal{C}^T=-\mathcal{C}\,,
$$

where $\mathcal{C}$ is the charge conjugation matrix and the Minkowski metric η_{AB} is almost positive. Furthermore we have introduced the following quantities:

$$
\begin{aligned}
&\widehat{\Gamma}_A=(-\Gamma_a\mathcal{I},\Gamma_{a'}\mathcal{J})\,,\quad \widehat{\Gamma}_{AB}=(-\Gamma_{ab}\mathcal{I},\Gamma_{a'b'}\mathcal{J})\,,\qquad \mathcal{I}=\Gamma^{01234}\,,\quad \mathcal{J}=\Gamma^{56789}\,,\\
&Q_Ih_+=Q_I\,,\qquad\qquad h_+=\frac{1}{2}(1+\Gamma_{11})\,,\qquad \Gamma_{11}=\Gamma_{01\cdots 9}\,.
\end{aligned}
$$

Now the supervielbeins $\mathbf{L}^A$ and L^α, and super spin connection $\mathbf{L}^{AB}$ can be read from the left-invariant Cartan one-form defined by

$$
g^{-1}dg = \mathbf{L}^AP_A+\frac{1}{2}\mathbf{L}^{AB}J_{AB}+Q_\alpha L^\alpha\,,
$$

where g is an element of the supercoset (A.3) and it is parametrized as

$$
g=g_xg_\theta\,,\quad g_\theta=\mathrm{e}^{Q\theta}\,,\quad Q=(Q_1,Q_2)\,,\quad \theta=\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}\,,
$$

and g_x and g_θ are the bosonic and the fermionic elements, respectively. Here we note that g_x should satisfy, by definition, that

$$
g_x^{-1}dg_x = e^AP_A+\frac{1}{2}\omega^{AB}J_{AB}\,,
$$

where e^A and ω^{AB} are the vielbein and the spin connection of $\mathrm{AdS}_5\times\mathrm{S}^5$.

After some algebra, we finally obtain the explicit expressions of the supervielbeins and super spin connection as follows[8]

$$\begin{aligned}
\mathbf{L}^A &= e^A + 2i\sum_{n=1}^{\infty}\bar{\theta}\Gamma^A\frac{\mathcal{M}^{2n-2}}{(2n)!}D\theta = e^A + 2i\bar{\theta}\Gamma^A\left(\frac{\cosh\mathcal{M}-1}{\mathcal{M}^2}\right)D\theta\,,\\
L^\alpha &= \sum_{n=0}^{\infty}\frac{\mathcal{M}^{2n}}{(2n+1)!}D\theta = \frac{\sinh\mathcal{M}}{\mathcal{M}}D\theta\,,\\
\mathbf{L}^{AB} &= \omega^{AB} - 2i\alpha\bar{\theta}\widehat{\Gamma}^{AB}i\sigma_2\sum_{n=1}^{\infty}\frac{\mathcal{M}^{2n-2}}{(2n)!}D\theta = \omega^{AB} - 2i\alpha\bar{\theta}\widehat{\Gamma}^{AB}i\sigma_2\frac{\cosh\mathcal{M}-1}{\mathcal{M}^2}D\theta
\end{aligned}$$

with

$$\begin{aligned}
\mathcal{M}^2 &= i\alpha\left(\widehat{\Gamma}_A i\sigma_2\theta\,\bar{\theta}\Gamma^A - \frac{1}{2}\Gamma_{AB}\theta\,\bar{\theta}\widehat{\Gamma}^{AB}i\sigma_2\right)\,,\\
D\theta &= d\theta + \frac{\alpha}{2}e^A\widehat{\Gamma}_A i\sigma_2\theta + \frac{1}{4}\omega^{AB}\Gamma_{AB}\theta\,.
\end{aligned}\tag{A.5}$$

By construction these should satisfy the following MC equations:

$$\begin{aligned}
d\mathbf{L}^A &= -\eta_{BC}\mathbf{L}^{AB}\mathbf{L}^C + i\bar{L}\Gamma^A L\,,\\
d\mathbf{L}^{ab} &= -\alpha^2\mathbf{L}^a\mathbf{L}^b - \eta_{cd}\mathbf{L}^{ca}\mathbf{L}^{bd} - i\alpha\bar{L}\widehat{\Gamma}^{ab}i\sigma_2 L\,,\\
d\mathbf{L}^{a'b'} &= +\alpha^2\mathbf{L}^{a'}\mathbf{L}^{b'} - \eta_{c'd'}\mathbf{L}^{c'a'}\mathbf{L}^{b'd'} - i\alpha\bar{L}\widehat{\Gamma}^{a'b'}i\sigma_2 L\,,\\
dL^\alpha &= -\frac{\alpha}{2}\mathbf{L}^A\widehat{\Gamma}_A i\sigma_2 L - \frac{1}{4}\mathbf{L}^{AB}\Gamma_{AB}L\,,
\end{aligned}\tag{A.6}$$

which are equivalent to the $psu(2,2|4)$ algebra (A.4).

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